Mechanics 1 January 2009 Solutions (OCR 4728

①

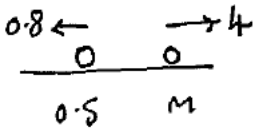
i) Conservation of momentum:

$(6 \times 0 \cdot 5) + 0 = (0 \cdot 8 \times 0 \cdot 5) + 4m$

$3 = 0 \cdot 4 + 4m$

$2 \cdot 6 = 4m$, $\underline{m = 6 \cdot 5 \text{ kg}}$

ii) After:



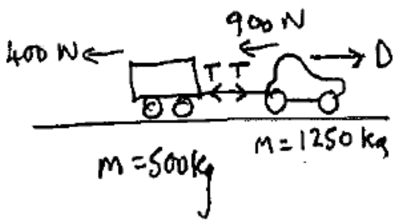
Conserve momentum →+

$(6 \times 0 \cdot 5) = 4m - (0 \cdot 8 \times 0 \cdot 5)$

$3 = 4m - 0 \cdot 4$

$3 \cdot 4 = 4m$, $\underline{m = 0 \cdot 85 \text{ kg}}$

②



i) Constant speed, so $\underset{\sim}{a} = 0$

No resultant force if no acceleration

D = Driving force

Consider car + trailer together & resolve horizontally

$400 + 900 = D$

$\underline{\text{Driving force} = 1300 \text{ N}}$

Consider just trailer: $\underline{T = 400 \text{ N}}$

ii) $\underset{\sim}{a} = 0 \cdot 6 \text{ ms}^{-2}$ →→

Newtons 2nd law $F = ma$

Car + trailer: $D - 1300 = (1250 + 500)\, 0 \cdot 6$

$D - 1300 = 1050$

$\underline{D = 2350 \text{ N}}$

Consider just trailer:

$T - 400 = ma$

$T - 400 = 500 \times 0 \cdot 6$

$T - 400 = 300$

$\underline{T = 700 \text{ N}}$

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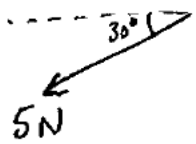
(3)



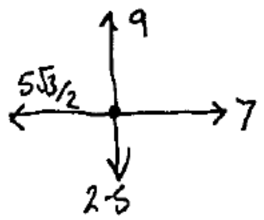
i) Component along x-axis has magnitude $5\cos 30°$

$= \frac{5\sqrt{3}}{2} = 4.33\text{ N}$

Component along y-axis has magnitude $5\sin 30°$ or $5\cos 60°$

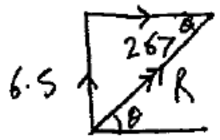
$= 2.5\text{ N}$

ii) Magnitude of resultant:



Resolve in y direction: $9 - 2.5 = 6.5\text{ N}\uparrow$

Resolve in x-direction $7 - \frac{5\sqrt{3}}{2} = 2.67\text{ N} \rightarrow$



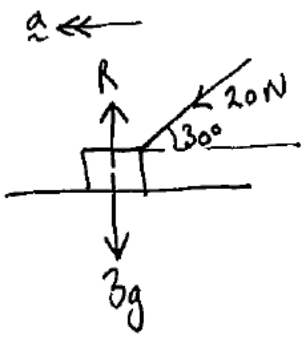
$|R| = \sqrt{6.5^2 + 2.67^2}$

$|R| = 7.03\text{ N}$

$\tan\theta = \left(\frac{6.5}{2.67}\right)$ $\theta = 67.6°$ with positive x-axis.

(4)

i)



Block will accelerate horizontally with a given by Newton II: $F = ma$

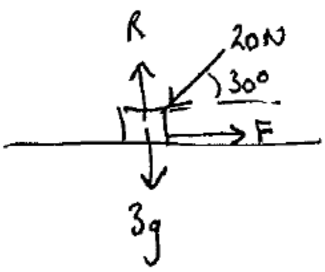
F is resultant horizontal force causing a,

So $F = 20\cos 30°$

$m = 3$

$a = \frac{20\cos 30°}{3} = 5.77\text{ ms}^{-2}$

ii)



Limiting equilibrium so $F = \mu R$ $\mu = F/R$ ①

Resolve(↑): $R = 3g + 20\sin 30° = 39.4$

Resolve (↔): $F = 20\cos 30°$

① $\mu = \frac{20\cos 30°}{(3g + 20\sin 30°)} = 0.44$

⑤

$t=0$, $u = 13\,ms^{-1}$

$\bullet\!\!\longrightarrow$ $\quad \underset{\sim}{a} = 0.8t\ ms^{-2}$ for t between 0 & 6.

A

$t=0$

i) Speed of Car when $t=6$: $\quad V = \int a\,dt$ as $\underset{\sim}{a}$ is a function of t

$$V = \int 0.8t\,dt$$

$$V = \frac{0.8t^2}{2} + c$$

When $t=0$ $\quad V=13$ $\quad$ So $13 = c$

$$\therefore V = 0.4t^2 + 13$$

When $t=6$, $\quad V = 0.4(6)^2 + 13$

$$\underline{\underline{V = 27.4\,ms^{-1}}}$$

ii) Want S when $t=6$:

$$S = \int v\,dt$$

$$S = \int (0.4t^2 + 13)\,dt$$

$$S = \frac{0.4t^3}{3} + 13t + k$$

When $t=0$, $S=0$, So $k=0$

When $t=6$ $\quad S = \frac{0.4(6)^3}{3} + (13\times 6)$

$$\underline{\underline{S = 106.8\,m}}$$

iii) x will increase as a Cubic function of t

V will increase as a Square function of t, so gradient of graph will always be increasing, and will not start at 0 – So not fig 3 or fig 1

a) Figure 2 could represent the motion.

b) Fig 1 has zero gradient at start so $V=0$, but this is not the case

Fig 3 has a gradient that decreases instead of increases.

Q6

i) Parcel falls 2.5m vertically. Want t & V:

$s = 2.5$
$u = 0$
v
$a = 9.8$
t

a) $s = ut + \frac{1}{2}at^2$

$2.5 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$2.5 = 4.9t^2$

$t^2 = \frac{2.5}{4.9}$

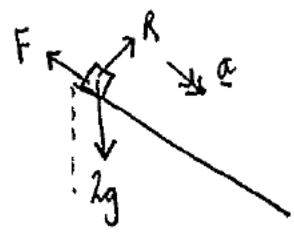
$t = \pm\sqrt{\frac{2.5}{4.9}} = \frac{5}{7}$ secs

b) $V^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 9.8 \times 2.5$

$|V| = 7\,ms^{-1}$

ii)



Parcel will accelerate down ramp. Need this a :

$F = \mu R$ as parcel moving

Resolve($\perp$): $2g\cos 30° = R$

Force down slope is $2g\cos 60° - F$

$= 2g\cos 60° - \mu R$

$= 2g\cos 60° - \mu\, 2g\cos 30°$

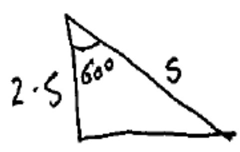
$= g - 0.2 \times 2g\frac{\sqrt{3}}{2}$

$= 6.4$

Newton II "F = Ma"

So $a = F/M = \frac{6.4}{2} = 3.2\,ms^{-2}$

Distance parcel moves down slope



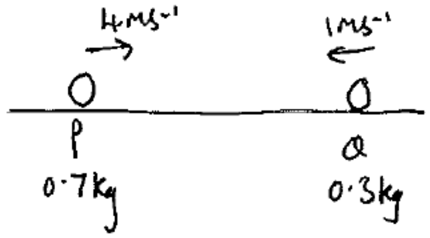
$\cos 60 = \frac{2.5}{s}$

$s = \frac{2.5}{\cos 60} = 5\,m$

$s = 5m$
$u = 0$
v
$a = 3.2$

$V^2 = u^2 + 2as$ $\quad V^2 = 2 \times 3.2 \times 5$ $\quad V = 5.66\,ms^{-1}$

(7)

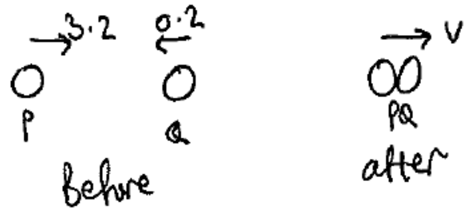


P's motion to collision: s = ?
u = 4
v want
a = −0.4
t = 2

$v = u + at$
$= 4 - 0.4 \times 2$
$V_P = 3.2\ ms^{-1}$ before collision

Q's motion to collision:
s = ?
u = 1
v want
a = −0.4
t = 2

$v = u + at$
$= 1 - 0.4 \times 2$
$V_Q = 0.2\ ms^{-1}$ before collision



Conservation of momentum: → +

$0.7 \times 3.2 - 0.3 \times 0.2 = 1 \times v$

$2.18\ ms^{-1} = v$ after collision

ii) 3 secs to collision:

Consider P:
s =
u = 4
v
a = −0.4
t = 3

$v = u + at$
$= 4 - 0.4 \times 3$
$v = 2.8\ ms^{-1}$

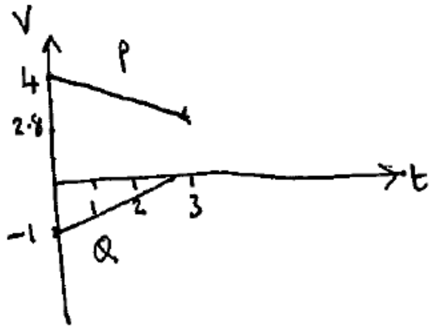
Consider Q:
s =
u = 1
v
a = −0.4
t = 3

$v = u + at$
$= 1 - 0.4 \times 3$ Negative
So Q stops before 3 seconds

When Q stops:
s =
u = 1
v = 0
a = −0.4
t =

$v = u + at$
$0 = 1 - 0.4t$
$0.4t = 1$
$t = \frac{1}{0.4} = 2.5$ secs

a)



b) Distance between them:

For P: $s = ut + \frac{1}{2}at^2$
$= 4 \times 3 - \frac{1}{2} \times 0.4 \times 3^2$
$= 10.2\ m$

For Q: $s = ut + \frac{1}{2}at^2$
$= 1 \times 2.5 - \frac{1}{2} \times 0.4 \times 2.5^2$
$s = 1.25\ m$

Total distance apart $= 10.2 + 1.25 = 11.45\ m$